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An Accurate Determination of Dielectric Loss Effect in Monolithic Microwave Integrated Circuits Including Microstrip and Coupled Microstrip Lines

D. MIRSHEKAR-SYAHKAL

Abstract —**For the first time, by a rigorous analysis, the performance of MIC planar transmission lines with lossy substrates can be studied accurately. The general structural shape chosen for the analysis includes infinitely thin metallic strips embedded within the layers of homogeneous dielectric substrates. The rigor of the analysis was guaranteed by the assumption of the propagation of an electromagnetic hybrid wave (i.e., TE+TM) along the planar transmission lines. An efficient computation was, however, achieved by implementing the spectral domain approach as the basis for the analysis. To test the analysis, phase constants, characteristic impedances, and attenuations, due to dielectric losses, were computed for microstrip and coupled microstrip lines. The results obtained were compared with those given previously by the spectral domain analysis in which dielectric losses were not included directly [1]. The comparison showed an excellent agreement between the two theories for low-loss substrates. However, for lossy substrates the present method is more accurate.**

I. Introduction

A new generation of microwave integrated circuits, the so-called "Monolithic Microwave Integrated Circuits" (MMIC), is under development. In MMIC, the aim is to integrate as many passive and active microwave components as possible on one single chip, in order to achieve the highest degree of compactness. This, of course, would be of great advantage where a large number of repeated circuits is required. Examples of this can be found, for instance, in active arrays of antenna.

Substrates used for MMIC are of the semiconductor type [2]. This is because the substrate should provide a ground for the fabrication of the active elements, as well as the passive components. Propagation of the electromagnetic waves through a semiconductor medium is usually subject to a large attenuation [3]. The loss of propagating energy in a semiconductor substrate is mainly due to the finite resistivity of the medium. For example, silicon can have resistivity varying between 100–1200 $\Omega \cdot$cm [3]. Therefore, substrates used in MIC with similar resistivities could dissipate energy equal to or greater than the energy dissipated in metallic parts, i.e., strips.

So far, the capability of the theories developed for computing the dielectric loss of planar transmission lines have been limited either by the assumption of the quasi-TEM waves propagation [4], [5], or, in some cases, by the very crude plane-wave approximation [6]. The first accurate analysis of dielectric loss in which the effect of dispersion was considered was introduced by Mirshekar-Syahkal and Davies [1], [7]. In this general analysis, a perturbation technique was developed in order to deal with planar structures with multidielectric layers leading to

$$\alpha_d = \frac{\omega \sum_{i=1}^{I} \epsilon_i \tan\delta_i \iint_{S_i} |\vec{E}_0|^2\, ds}{2\,\mathrm{Re} \iint_S \vec{E}_0 \times \vec{H}_0^* \cdot d\vec{s}} \tag{1}$$

where $\vec{E}_0$ and $\vec{H}_0$ are the unperturbed (zero-loss) fields. In (1), I denotes the number of dielectric layers, ϵ_i, $\tan\delta_i$ and S_i are the i^{th} dielectric layer permittivity, loss tangent, and cross-sectional area, and ω and S represent the angular frequency and the total transmission-line cross-sectional area, respectively. $\vec{E}_0$ and $\vec{H}_0$ are determined through the generalized spectral domain technique. A computer program providing $\vec{E}_0$ and $\vec{H}_0$, and subsequently α_d, is already available [8].

Though expression (1) can adequately describe the attentuation of a mode along a planar transmission line for substrates with small $\tan\delta$, the accuracy of (1) is not known. Especially in very lossy dielectric substrates, it is not only the accuracy of α_d that is important, but the effect of dielectric loss on wave length and on characteristic impedance can also be of considerable significance, particularly where the coupling of two lines becomes a point of interest.

To alleviate the shortcomings of earlier theories, the generalized method developed in [1], [7] is further extended to include *a priori* a complex dielectric constant. This new extension of the generalized spectral domain technique is then examined by solving two common structures, microstrip and coupled microstrip, taking different loss values for their substrates. Through this examination, the accuracy of the perturbation equation (1) can be examined.

II. Theory

A generic cross section of an arbitrary multistrip multidielectric MIC planar transmission line is shown in Fig. 1(a). The metallic enclosure is the inevitable packaging cover, and, therefore, its effect on the propagation of the wave has to be counted in the analysis. It is assumed that the dielectrics are homogeneous and the thickness of the strips satisfies the relation

$$\text{skin depth} \ll \text{strip thickness} \ll \text{dielectric thickness}.$$

A good metallization for the strips, and the use of a good conductor for the enclosure, allows the assumption of a perfect conductor for the metallic parts. However, the loss due to imperfect conductors can be calculated through a perturbation expression given in [1]. For brevity, conductor loss analysis is excluded from the following theory.

Considering the dielectric layers, the dielectric loss for each homogeneous dielectric region can be represented by the imaginary parts of a complex dielectric constant, given by

$$\epsilon_i = \epsilon_i' - j\epsilon_i'' = \epsilon_i'(1 - j\tan\delta_i)$$

where

$$\tan\delta_i = \epsilon_i''/\epsilon_i'.$$

In the case of any loss-free dielectric, of course, $\epsilon_i = \epsilon_i'$ and $\tan\delta_i = 0$.

Due to the mixed dielectric boundary of the problem, an accurate analysis without the assumption of an electromagnetic hybrid wave (i.e., TE+TM) is not possible. Based on this rigor-

Manuscript received April 21, 1983; revised June 20, 1983.
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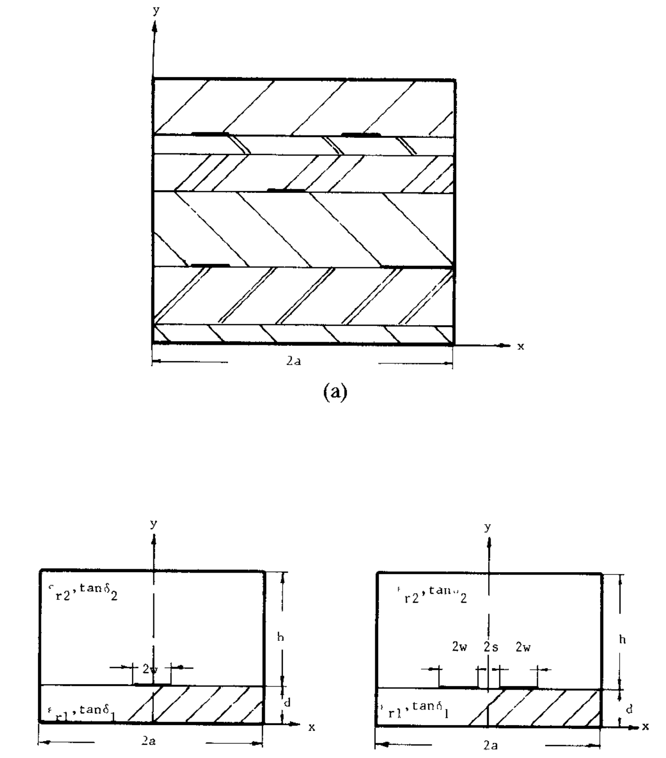

Fig. 1. (a) A general multidielectric multistrip planar structure. (b) Microstrip on lossy substrate, $2w = d = 0.5$ mm, $h = 19.5$ mm, $a = 10$ mm, $\tan\delta_2 = 0$, $\epsilon_{r2} = 1$ and $\epsilon_{r1} = 10$. (c) Coupled microstrip on lossy substrate, $2w = d = 0.5$ mm, $h = 19.5$ mm, $a = 10$ mm, $s = 0.1$ mm, $\tan\delta_2 = 0$, $\epsilon_{r2} = 1$, and $\epsilon_{r1} = 10$.

ous assumption, several techniques, developed to solve various loss-free planar transmission-line problems, are already available [7]. Due to the advantages offered, the generalized spectral domain approach will be extended in this paper [1], [7], [9].

Work on the spectral domain method with many examples can be found in [7], and so, to economize on space, in the following some details are omitted.

To start with, a complex propagation constant

$$\Gamma = \alpha + j\beta$$

is introduced where α represents the attenuation constant due to the overall dielectric losses and β denotes the phase constant. Obviously, in the absence of dielectric loss $\alpha = 0$, and the problem turns into the form previously discussed in full in [1], [7].

The propagation of a hybrid mode along the line requires nonzero longitudinal fields. These fields in each dielectric region can be obtained by means of two independent potential functions through expressions [7], [9]

$$E_{z,i} = -\frac{k_i^2 + \Gamma^2}{\Gamma}\psi_i^{(e)}(x,y)e^{-\Gamma z} \tag{2}$$

$$H_{z,i} = -\frac{k_i^2 + \Gamma^2}{\Gamma}\psi_i^{(h)}(x,y)e^{-\Gamma z} \tag{3}$$

where

$$k_i^2 = \omega^2\mu_i\epsilon_i$$

and i represents the i^{th} dielectric region. Equations (2) and (3) are similar to those expressed for the loss-free condition in [1], [7], the only difference being in the complex propagation constant. Thus the finite Fourier transform [7]

$$\tilde{\psi}_i^{(e,h)} = \int_{-a}^{+a}\psi_i^{(e,h)}(x,y)e^{j\alpha_n x}\,dx \tag{4}$$

can be performed to reduce the two-dimensional Helmholtz equation into the one-dimensional equation.

$$\frac{\partial^2}{\partial y^2}\tilde{\psi}_i^{(e,h)} - \gamma_{i,n}^2\tilde{\psi}_i^{(e,h)} = 0 \tag{5}$$

where

$$\gamma_{i,n}^2 = \alpha_n^2 - \Gamma^2 - k_i^2$$

and α_n in (4) is generally expressed by

$$\alpha_n = n\pi/2a, \qquad n = 0,1,2,\cdots$$

where $2a$ is the enclosure width. In (5), $\gamma_{i,n}$ is a complex function and it turns into a pure imaginary or real function if the dielectrics are considered lossless [1], [7].

The solution to (5) is straightforward and can be followed as quoted in [7]. However, it is worth noting that, in this problem, the arguments of sinh and cosh appearing in the solution of (5) are complex.

Having derived the potential functions $\tilde{\psi}_i^{(e,h)}$ in the spectral domain, one can now find all the field components in this domain. The use of boundary conditions at the dielectric interfaces results in a system of equations in the Fourier domain relating the x and z components of electric fields and currents. A matrix representation of these equations is given by

$$[G][\tilde{J}] = [\tilde{E}] \tag{6}$$

where $[\tilde{J}]$ and $[\tilde{E}]$ are the matrices of currents and electric fields at the interfaces between the dielectrics and the metallic strips. For transmission lines with a single layer of strips and several layers of dielectrics, (6) reduces to

$$\begin{bmatrix} G_{1,1}(n,\Gamma) & G_{1,2}(n,\Gamma) \\ G_{2,1}(n,\Gamma) & G_{2,2}(n,\Gamma) \end{bmatrix} \begin{bmatrix} \tilde{J}_x \\ \tilde{J}_z \end{bmatrix} = \begin{bmatrix} \tilde{E}_z \\ \tilde{E}_x \end{bmatrix} \tag{7}$$

where $\tilde{E}_x$, $\tilde{E}_z$, $\tilde{J}_x$, and $\tilde{J}_z$ are the Fourier transforms of the fields and currents at the strip-to-dielectric interface. The $[G]$ matrix elements in (7) can be generated by a procedure described in [7], [10]. Notice that in loss-free conditions $[G]$ is a real matrix.

To obtain Γ from (7), fields or currents are expanded first in terms of some appropriate basis functions. Then Galerkin's method, together with Parseval's identity, are applied, resulting in a set of homogeneous equations whose nontrivial solution is Γ. Having found $\Gamma = \alpha + j\beta$, a reverse strategy obtains all the fields in the Fourier domain. Consequently, parameters such as characteristic impedance requiring a knowledge of fields can be easily computed.

III. Numerical Results

So far, in Section II a theory has been discussed which is very similar to that developed in [7], but is an extension of it in which the final matrix is now complex. The zero of the determinant of the final matrix, i.e., $\Gamma = \alpha + j\beta$, may be traced by various algorithms written to calculate the complex roots of a function. In the search for an efficient technique, the quadrature technique of Muller [11] seems very suitable. Since root deflation is performed by Muller's algorithm for functions of several roots, this technique is bound to lose accuracy as it embarks on finding a new root. It is possible, however, to compute the physical solution first. Alternatively, one can define a very stringent accuracy criterion for the first root, so that as the process of root-finding progresses, if the number of roots are not very large, the accuracy for the desired root would fall within the limit of the predefined accuracy. By taking precautions, the problem of error caused by the deflation procedure can be controlled.

The special examples treated by the theory presented in Section II are the microstrip and coupled microstrip, shown in Figs. 1(b) and 1(c), respectively, which both have considerable applications. In both cases, J_z is taken as the only current existing on the strip and the role of J_x is ignored. This approximation is the "zero-order solution," and would be adequate for many engineering problems, provided that the choice of J_z is close to the physical distribution. For microstrip, a function derived in [12] and examined by several authors [7], [9], which seems an adequate approximation for J_z, is given by

$$J_z = f(x) = \frac{1}{2w}\left(1 + \left|\frac{x}{w}\right|^3\right), \qquad |x| < w. \tag{8}$$

For the coupled-strip case, expression (8) is adopted for the current distribution on each strip and, depending upon the mode of operation, the following forms are valid:

$$J_z = \begin{cases} f(x - s - w), & s < x < s + 2w \\ f(x + s + w), & -s > x > -s - 2w \end{cases} \tag{9}$$

for the even mode and

$$J_z = \begin{cases} f(x - s - w), & s < x < s + 2w \\ -f(x + s + w), & -s > x > -s - 2w \end{cases} \tag{10}$$

for the odd mode. By definition, in a coupled microstrip, the mode is called even when E_z and H_z are even, and odd functions of x, respectively, and the mode is called odd when E_z and H_z are odd and even functions of x, respectively. Care should be taken, however, in the coupled strip configuration that the functions introduced by (9) and (10) remain acceptable when the strips are not very close. Otherwise, either a better physical representation of the currents is needed, or the currents have to be approximated through a set of basis functions [1], [7]. In the approximate form for J_z, singularities due to the edge conditions are not included. In fact, in the zero-order solution, a very accurate treatment of the edge condition, does not contribute significantly to the accuracy of the solution. By contrast, calculation of the conductor loss is very sensitive to the function representing the edge condition [1], [7].

We wish to observe the effect of a lossy dielectric on the characteristic impedance. It is customary to choose the impedance definitions

$$Z = 2P/I_z^2 \tag{11}$$

$$Z_{e,o} = P/I_z^2 \tag{12}$$

as the characteristic impedance for the microstrip and the coupled microstrip, respectively [1]. Subscripts e and o refer to even and odd modes of operation. I_z in (11) and (12) is the total current on a strip given by

$$I_z = \int_{-w}^{+w} I_z(x)\, dx \tag{13}$$

and P is the transmitted power given by

$$P = \frac{1}{2}\,\mathrm{Re}\iint_S \vec{E} \times \vec{H}^* \cdot d\vec{s} \tag{14}$$

where S is the cross-sectional area of the line.

Based on the theory of Section II and the approximation made in this Section, a computer program has been developed to compute attentuation due to the dielectric loss, phase constant, and characteristic impedance of microstrip and coupled microstrip. Some results produced by this program are shown in Figs. 2–5, where $\tan\delta_1$ is assumed to be a parameter decreasing from a large value 1.0 to a small value 2×10^{-4}. In the given examples, it is assumed that the real part of the dielectric permittivity is constant; $\epsilon_1' = \epsilon_{r1}\cdot\epsilon_0 =$ Constant. This is not, however, an unrealistic assumption. For instance, in silicon substrates, as reported in [3], ϵ_{r1} remains unchanged over the microwave frequency range. In semiconductor substrates, due to the dominant nature of the ohmic loss, the relationship between the loss tangent and the frequency is approximately given by

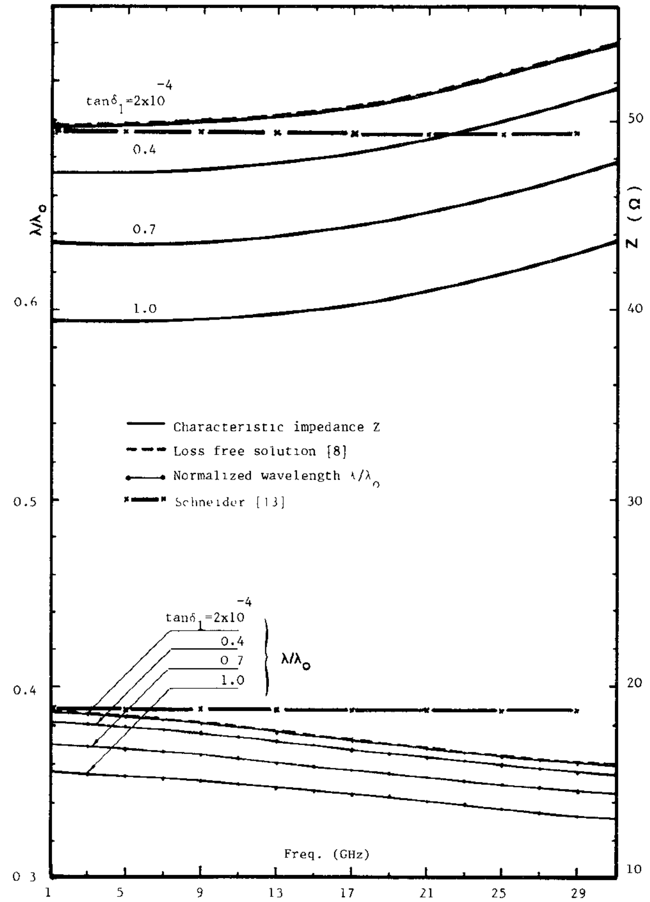

Fig. 2. Characteristic impedance and normalized wavelength of the fundamental mode of the microstrip shown in Fig. 1 (b).

$$\tan\delta_i \simeq \frac{\sigma_i}{\omega\epsilon_i'}$$

where σ_i is the material conductivity of the ith layer. Therefore, to obtain any parameter from Figs. 2–5, the $\tan\delta_1$ corresponding to the frequency of operation first must be determined.

Examination of λ/λ_0 in Figs. 2, 3, and 4 shows that the normalized wavelength asymptotically approaches that obtained under the loss-free condition (broken line). As would be expected, for $\tan\delta_1 < 0.1$, the normalized wavelengths were found to be virtually equal to those obtained by the computer programs ZERO 1 described in [8].

The curves in Figs. 2, 3, and 4, representing the characteristic impedances, illustrate the same behavior as explained for the normalized wavelengths. In these figures, a decrease in the characteristic impedance values with increase in $\tan\delta_1$ is apparent. This phenomenon has indeed been observed in experimental studies on silicon used as a microstrip substrate [3].

The dielectric loss for the microstrip in Fig. 5 demonstrates the close relationship existing between the present technique and the perturbation method (1) at small values of $\tan\delta_1$. Very similar

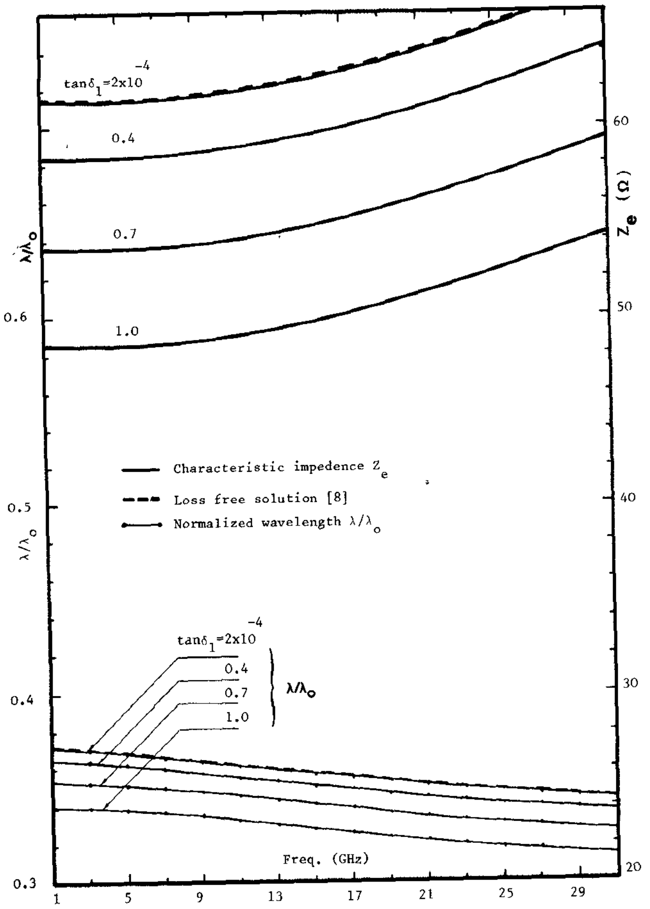

Fig. 3. Characteristic impedance and normalized wavelength of the even mode of the coupled microstrip shown in Fig. 1 (c).

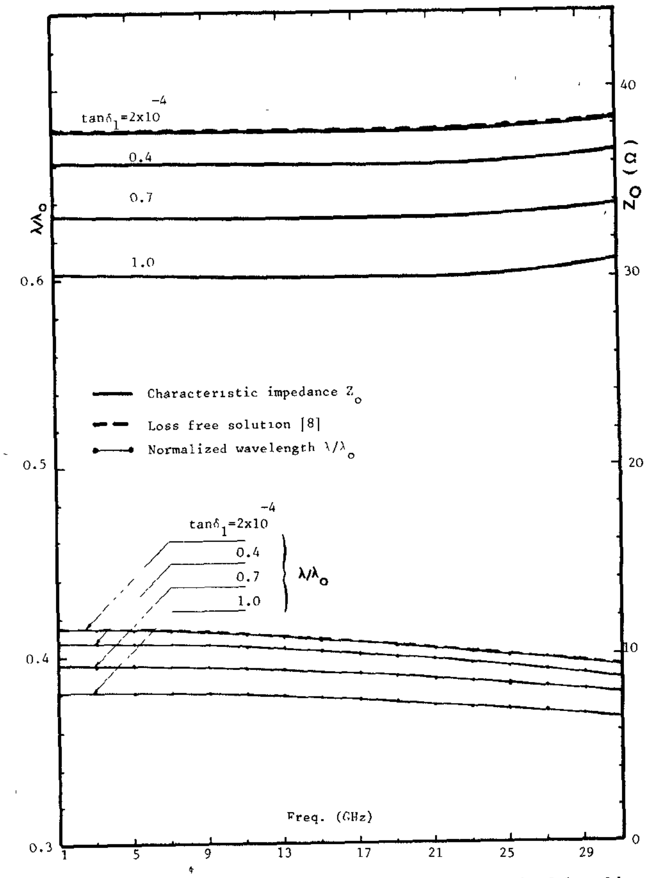

Fig. 4. Characteristic impedance and normalized wavelength of the odd mode of the coupled microstrip shown in Fig. 1 (c).

results were also achieved for the coupled microstrips which are not shown in this paper.

Computations showed that, for substrates with $\tan\delta_1 < 0.1$, the perturbation method can be satisfactorily used for the analysis of dielectric loss in the planar transmission lines fabricated from low-loss dielectrics.

As a further comparison, parameters λ/λ_0, Z, and α_d for the microstrip line were obtained by Schneider's quasi-TEM technique [4], [13] and displayed in Figs. 2 and 5. As expected, at low frequency and for small $\tan\delta_1$, the quasi-TEM results are in good agreement with the present technique results. However, at high frequency or for large $\tan\delta_1$, the quasi-TEM method cannot be regarded as accurate.

IV. Conclusions

Using the generalized spectral domain technique, a rigorous analysis for the effect of dielectric loss in planar transmission lines with lossy substrates is given. In this analysis, a complex propagation constant is obtained with the usual phase and attenuation components. As the test examples, a microstrip and a coupled microstrip were treated by the theory developed. The computed results for the wavelength and the characteristic impedance were compared with those obtained through the spectral domain technique developed for the lossless planar transmission lines. The dielectric losses were also contrasted with those computed by means of a perturbation method. These comparative studies showed that, for substrates of small dielectric loss (around $\tan\delta_1 < 0.1$), the theories were in excellent agreement. However,

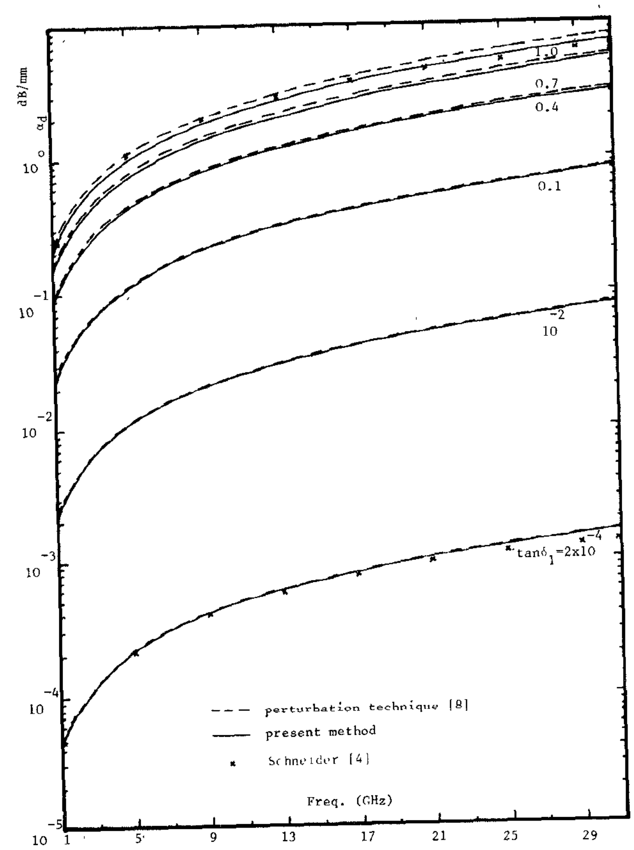

Fig. 5. Dielectric loss of the fundamental mode of the microstrip shown in Fig. 1 (b).

for lossy dielectric substrates, the theory developed in this paper has better accuracy and is clearly preferred.

In the results obtained for the microstrip and the coupled microstrip, the lowest order of accuracy, "zero order", was considered for the solutions. Nevertheless, it is not too difficult to achieve a higher degree of accuracy by increasing the order of the final matrix. This can be accomplished by simply expanding the currents or the fields in terms of appropriate basis functions like those, for instance, in [1].

The computer programs developed to generate the quoted results are in FORTRAN and are available by request.

Acknowledgment

The author wishes to thank Dr. J. B. Davies for his invaluable comments in the course of preparation of this paper.

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Mathematical Representation of Microwave Oscillator Characteristics by Use of the Rieke Diagram

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***Abstract*—This paper shows that the characteristics of oscillators can be phenomenologically expressed by a polynomial function of frequency and amplitude, provided the output signal is nearly sinusoidal, especially at microwave frequency. A method is presented of determining the coefficients of the polynomial from several points on the Rieke diagram, with two examples being shown. The characteristics of oscillators can consequently be represented by several parameters, as in the case of electron tubes and transistors, so that the design of an oscillator circuit may become easier with the aid of an electronic computer.**

Manuscript received December 16, 1982; revised June 17, 1983. This work was supported by a Grant from the Nippon Telegraph and Telephone Public Corporation, Japan

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I. Introduction

So far, extensive studies have been performed on oscillator characterization. An important contribution was made by van der Pol. Since then, almost all studies on oscillators have been based on his oscillator model. It appears, however, that few have investigated the oscillator model itself.

The purpose of this paper is to provide a new mathematical oscillator model. It will be shown that the nonlinear admittance of oscillators can be phenomenologically expressed by a polynomial function of frequency and amplitude, provided the output signal is nearly sinusoidal, especially at microwave frequency. A method is presented concerning how to determine the coefficients of the polynomial from several points on the Rieke diagram, and two examples will be shown. Although much time is consumed to draw diagrams, the Rieke diagram has so far been used to express the characteristics, especially of microwave oscillators [1], [2]. This is because the Rieke diagram has advantages since the equi-power and equi-frequency loci are geometrically plotted on the Smith chart, and since the load admittance is represented within a circle of finite extent.

In this paper, a mathematical expression of oscillator characteristics is proposed instead of the geometrical expression, so that the characteristics of oscillators may be represented by several parameters, as in the case of electron tubes and transistors. The design of oscillator circuits will then become easier with the aid of an electronic computer.

II. Mathematical Expression of Nonlinear Admittances

A. Van der Pol's Oscillator

Van der Pol gave a basic mathematical expression of oscillator characteristics allowing for nonlinearity. If his formulation is viewed from the standpoint of the fundamental oscillation frequency, the oscillator admittance can be represented by a function of frequency and voltage amplitude squared as [3]

$$Y(j\omega, |V|^2) = -G_0 + G_V|V|^2 + jB_\omega \Delta\omega \tag{1}$$

where $\Delta\omega = \omega - \omega_0$, and ω_0 is the center frequency.[1] As is explained in the next section, the Rieke diagram of (1) is represented in the form of Fig. 2(b), while that of an existing oscillator is of Fig. 9. These two Rieke diagrams are different from each other chiefly in the following ways: i) Load locus on which maximum output power is generated is a circle in the case of van der Pol's oscillator, while an existing oscillator produces the maximum power at a single point on the Rieke diagram. ii) The Rieke diagram of van der Pol's oscillator is symmetric, while that of an existing oscillator is usually asymmetric.

B. Generalization of Oscillator Admittance

Fig. 1 shows an equivalent circuit of coupling of a microwave oscillator to a load. The effect of coupling strength of the

[1]The oscillation frequency when a matched load is connected to the line.